AP Calculus AB Summer Assignment

Name:_____

In preparation for a successful year in AP Calculus AB, Complete this packet and show all your work. **You will have <u>a test</u> based on the content of this packet <u>Tuesday Aug 15th, 2023</u>** Mastery of these topics is important in this class. If you have difficulty with a topic, use your IM3 Honors notes, videos posted on google classroom, the internet (such as Khan Academy) and other resources to learn more. Every single question has a related video providing concepts as well as sample questions to help you. Please see the red note for the corresponding video for each question.

** Round answers to the nearest .001 (Third digit after decimal) except where exact answers are required. **

1. The graphs of these functions will be used routinely in the course. Accurately graph each function on the grids provided at the end of this packet and classify each function as even, odd or neither. Practice these graphs until you can demonstrate them from memory.

   a. $y = x$ b. $y = \sqrt[3]{x}$ c. $y = e^x$ d. $y = \cos x$ e. $y = x^2$ f. $y = \dfrac{1}{x}$

   g. $y = \dfrac{1}{x^2}$ h. $y = \tan x$ i. $y = x^3$ j. $y = |x|$ ~~k. $y = [x]$~~   l. $y = \cot x$

   m. $y = \sqrt{x}$ n. $y = \ln x$ o. $y = \sin x$ p. $y = \sec x$ q. $y = \csc x$ r. $y = \sqrt{9 - x^2}$
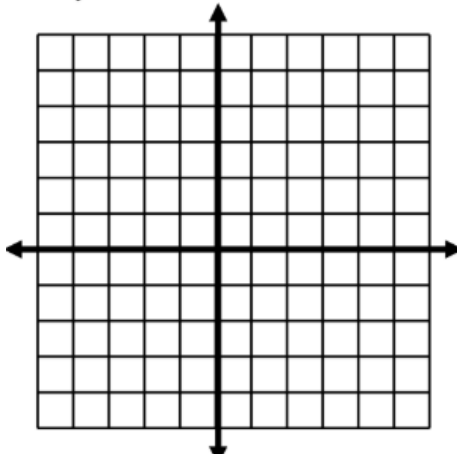
   * Remember to classify each function above as even, odd or neither.

** Answers are on the back. Any other questions, email Mrs. Chao by <u>gchao@psusd.us</u>

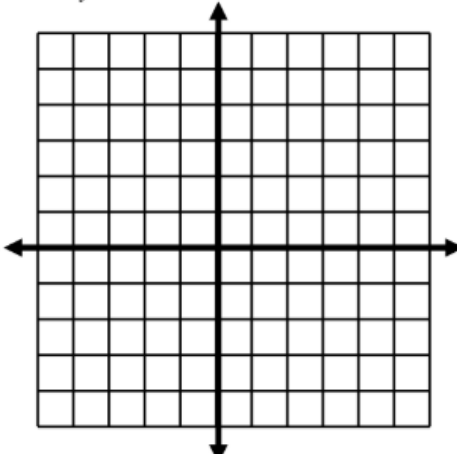**You may skip k, p, q**

a. $y = x$ Even/Odd/neither

b. $y = \sqrt[3]{x}$ Even/Odd/neither

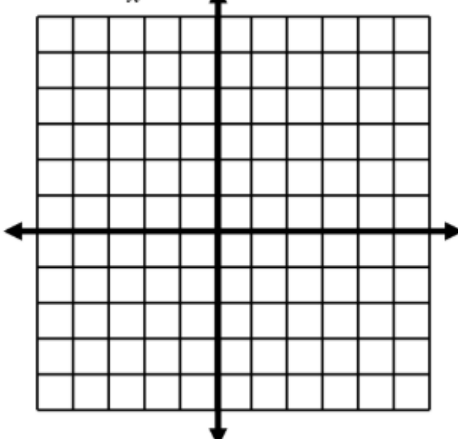c. $y = e^x$ Even/Odd/neither
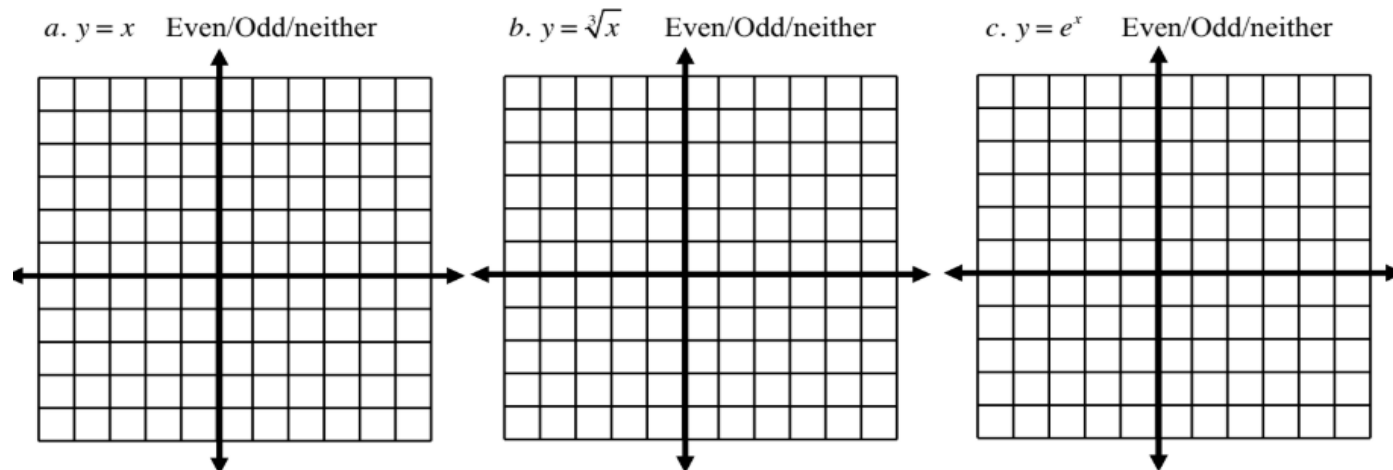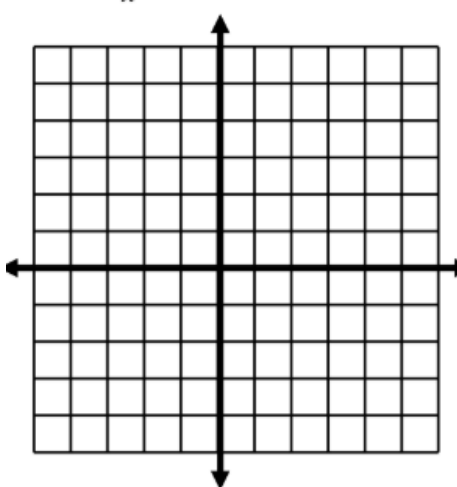
$d.\ y = \cos x$ Even/Odd/neither
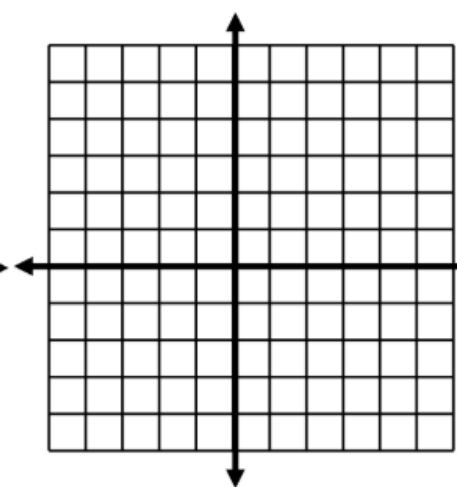
$e.\ y = x^2$ Even/Odd/neither

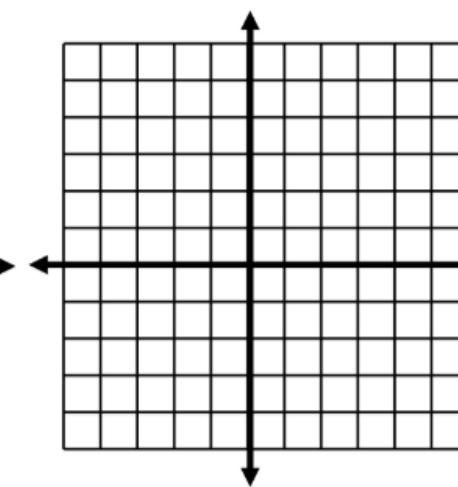$f.\ y = \dfrac{1}{x}$ Even/Odd/neither
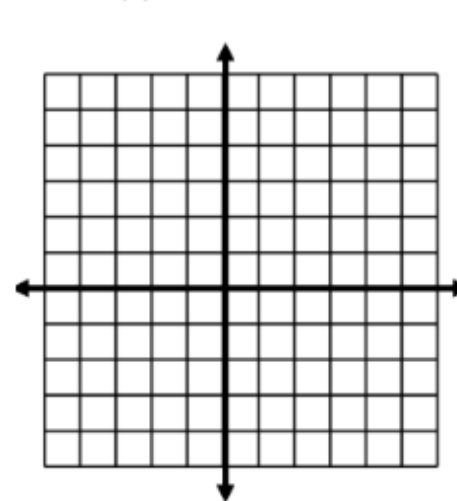
$g.\ y = \dfrac{1}{x^2}$ Even/Odd/neither

$h.\ y = \tan x$ Even/Odd/neither

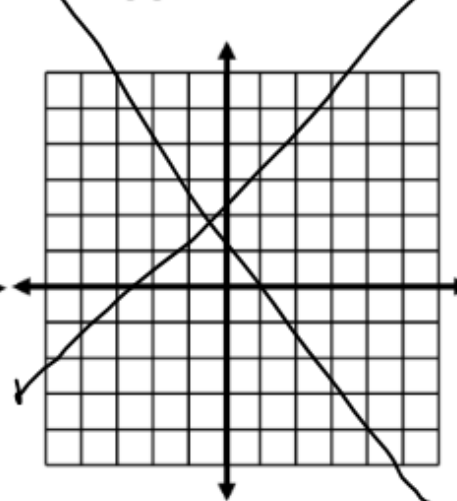$i.\ y = x^3$ Even/Odd/neither

$j.\ y = |x|$ Even/Odd/neither

$k.\ y = [x]$ Even/Odd/neither

$l.\ y = \cot x$ Even/Odd/neither

m. $y = \sqrt{x}$ Even/Odd/neither

n. $y = \ln x$ Even/Odd/neither

o. $y = \sin x$ Even/Odd/neither
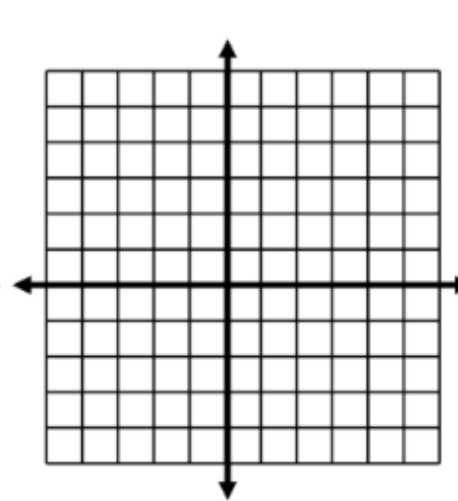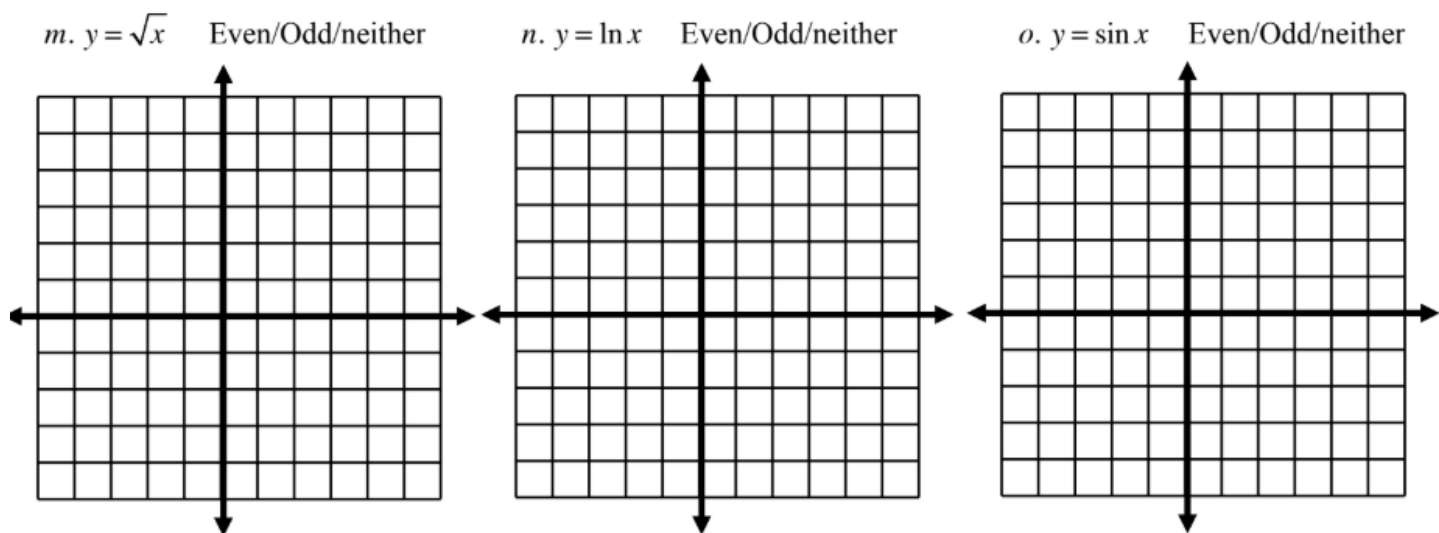
2.   There exists a function $y = f(x)$, such that $f(-.4) = -.864$ *and* $f(-.6) = -1.416$.

   a.   If *f(x)* is an even function, find $f(.4) =$ _____

   b.   If *f(x)* is an odd function, find $f(.6) =$ _____

Lines

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	the definition of slope
$y = mx + b$	slope-intercept form equation of a line
$Ax + By = C$	standard form equation of a line
$y - y_1 = m(x - x_1)$	point-slope form equation of a line
$x = k$	the equation of a vertical line
$m_1 = m_2$	the slopes of *parallel lines* are equal
$m_1 m_2 = -1$	the product of slopes of *perpendicular lines* is −1

Examples

Question: Find the equation of a line through the point (-2, 3) with a slope of 4.

Solution: Since we are given a point and a slope, use the point slope form.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 4(x + 2)$$

It is not necessary to simplify further.

Question: Find the equation of a line through the point (1, -5) that is parallel to the line $2x - 3y = 7$.

Solution: Parallel lines have the same slope so we must first find the slope of the line $2x - 3y = 7$. Solving for y yields

$$y = \tfrac{2}{3}x - \tfrac{7}{3}$$

The slope of this line is $\tfrac{2}{3}$ so the slope of the parallel line is also $\tfrac{2}{3}$. Using the point slope form,

$$y + 5 = \tfrac{2}{3}(x - 1)$$

Watch Topic 2 Video " Linear Functions" if you need help!!

3. a. Given $g(x) = -x^2 + 3$, write the equation of the line through $g(-2)$ and $g(3)$.

   (Use g(-2) and g(3) to find slope)

   b. Write the equation of the line that passes through the x-intercept of $2(3x - y) = -3$ and is perpendicular (normal) to this line.

4. Solve for x. **Give exact answers, not decimal.**

    a. $e^{2x} = 5$

                                            b. $2 + \cos^2 x = 3\sin^2 x$ $0 \le x < 2\pi$

    c. $\ln(x + 3) = 4$

Example

If $f(x) = x^2 - 5x + 8$,

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{(x+h)^2 - 5(x+h) + 8 - (x^2 - 5x + 8)}{h}$$

$$\frac{x^2 + 2xh + h^2 - 5x - 5h + 8 - x^2 + 5x - 8}{5}$$

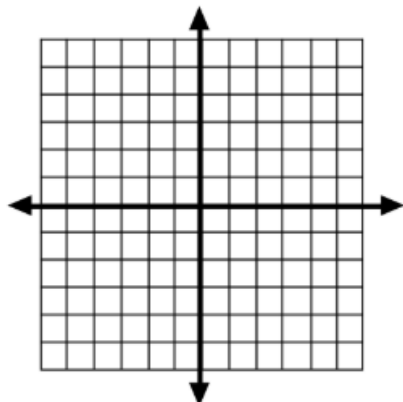$$\frac{h^2 + 2xh - 5h}{h} = \frac{h(h + 2x - 5)}{h} = h + 2x - 5$$

5. Given $f(x) = x^2 + 2x + 3$, $find$ $\frac{f(x+h) - f(x)}{h}$ This is also known as the average rate of change.

6. Graph:

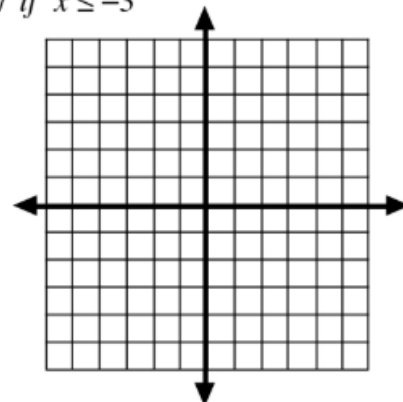a. $f(x) = 2|x+3| - 1$

b. $g(x) = \begin{cases} 2x+5 & \text{if } x > -3 \\ -2x-7 & \text{if } x \le -3 \end{cases}$

7. Graph the following function

a. $f(x) = \dfrac{x^2+x-12}{x^2+2x-15}$

b. $f(x) = \dfrac{x^2+x-12}{x-3}$

8. Expand using Log Properties:

a. $\ln\left(x^2\sqrt{y}\right)$

b. $\log_3\left(\dfrac{x+3}{x^2}\right)$

9. Given: $f(x) = 4\sin\left(\dfrac{\pi}{2}x\right) - 2$

    a. Amplitude _____

    b. Period _____

    c. Graph one period.

    d. Write the equation of the line through the maximum point and minimum point of the period graphed.

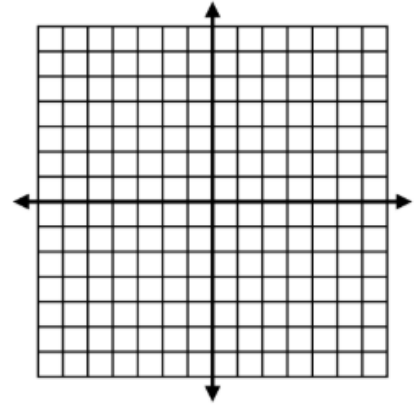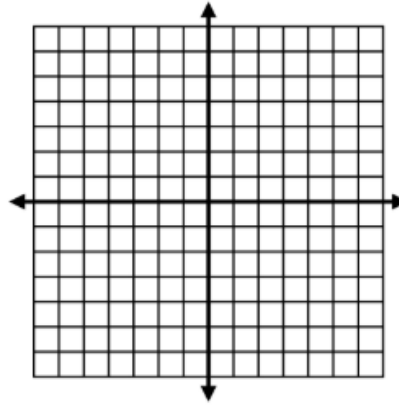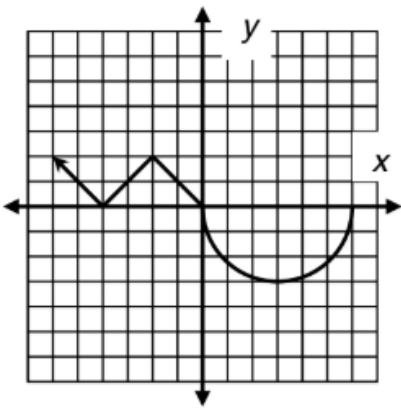10. Given the graph of $y = f(x)$ below: a. graph $y = -f(x)$ b. graph $y = f(-x)$

11. Given: $y = \dfrac{2(x+3)(x-2)}{(x-3)(x+3)}$

Identify:

a. any holes _____

b. any vertical asymptotes _____

c. any horizontal asymptotes_____

d. x - intercepts_____

e. y - intercepts _____

f. Graph.
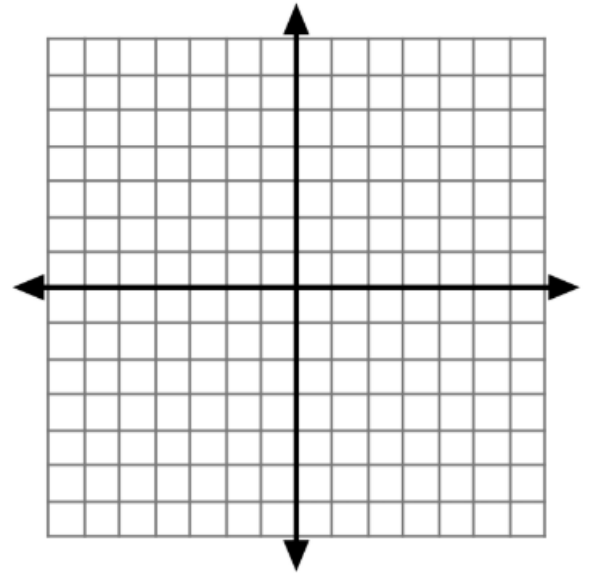
Q11 Watch Topic 6 Video "Rational Functions" for help
Watch Topic 12 Video "Geometry Related and All Others" if you need help

12. A **closed** box with a square base of side (x) and
    height (y) has a surface area of 100 sq. ft.

a. Express the volume of the box as a function
    of x . $V(x) =$
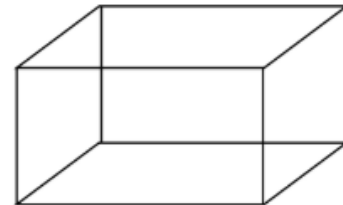
(Make the surface area = 100 first to get the relationship of x and y, then find the volume)

b. Domain of V(x) = _____   Range of V(x) = _____

13. Simplify

a. $\dfrac{x^3-125}{x-5}$

b. $\dfrac{x^3+27}{x^2-9}$

c. $\dfrac{\dfrac{1}{x+3}-\dfrac{1}{3}}{x}$

d. Rationalize the denominator and simplify:

$$\dfrac{x}{2-\sqrt{4-x}}$$

14. The area of the trapezoid at right is is $30\sqrt{2}\;\;cm^2$. Find the exact value of x.
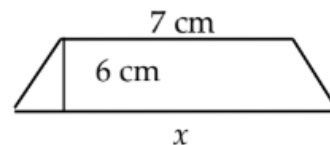
The formula for Trapezoid Area A = $\frac{1}{2}h\,(b1\,+\,b2)$, plug in all given number and variable and solve x

15. Water is poured into a conical container with a diameter of 20 cm and a height of 38 cm. If the volume of a cone is one-third the volume of a cylinder, write an equation for the volume of water as a function of the height of the water in the container.

16. Given $f(x) = 3x - 5$ and $g(x) = 2(x-4)^2 + 2x$, calculate the following.

    a.) $g(f(x))$ b.) $f(g(3))$ c.) $f^{-1}(x)$

Use the below trig identities above to simplify Q19

19. Simplify: a. $\dfrac{1 - \cos^2 x}{\sec^2 x - 1}$ b. $\sec x \cdot \sin 2x$

Important Trig Identities		
$\sin^2 x + \cos^2 x = 1$	$\tan^2 x + 1 = \sec^2 x$	$1 + \cot^2 x = \csc^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\cos 2x = 2\cos^2 x - 1$	$\cos 2x = 1 - 2\sin^2 x$
$\sin 2x = 2\sin x \cos x$	$\tan x = \dfrac{\sin x}{\cos x}$	$\cot x = \dfrac{\cos x}{\sin x}$

18. Evaluate:

  a. $\sin\left(\dfrac{2\pi}{3}\right)$ b. $\cos\left(-\dfrac{5\pi}{4}\right)$ c. $\tan\left(\dfrac{11\pi}{6}\right)$ d. $\csc\left(\dfrac{7\pi}{6}\right)$

19 Factoring Trinomials (**Watch Topic 1 Video "Factoring" Video if you need help**)

Factoring Trinomials: $ax^2 + bx + c$

$$2x^2 - 5x - 3 = (2x + 1)(x - 3)$$

1. $2x^2 - 5x - 3$
2. $3x^2 + 10x - 8$
3. $2y^2 + 15y + 7$
4. $7a^2 - 11a + 4$
5. $5n^2 + 17n + 6$
6. $4y^2 + 8y + 3$
7. $3x^2 + 4x - 7$
8. $2x^2 + 13x + 15$
9. $9y^2 + 6y - 8$
10. $6x^2 - 7x - 20$

11. $2n^2 - 3n - 14$
12. $5n^2 + 2n + 7$
13. $10x^2 + 13x - 30$
14. $12y^2 + 7y + 1$
15. $2n^2 + 9n - 5$
16. $2x^2 + 7x + 6$
17. $5a^2 - 42a - 27$
18. $15x^2 - 28x - 32$
19. $8a^2 - 10a + 3$
20. $2y^2 - 3y - 20$

20. Factoring the difference of two squares

Factoring the Difference of Two Squares

$$a^2 - 36 = (a + 6)(a - 6)$$
$$3x^2 - 48 = 3(x^2 - 16) = 3(x + 4)(x - 4)$$

1. $x^2 - 1$
2. $x^2 - 9$
3. $x^2 + 4$
4. $x^2 - 25$
5. $9y^2 - 16$
6. $4x^2 - 25$
7. $9x^2 - 1$
8. $a^2 - x^2$
9. $25 - m^2$
10. $x^2 - 16y^2$
11. $25m^2 - n^2$

12. $-x^2 + 16$
13. $36m^2 - 121$
14. $2x^2 - 8$
15. $25 + 4x^2$
16. $4a^2 - 81b^2$
17. $12x^2 - 75$
18. $a^2b - b^3$
19. $-98 + 2x^2$
20. $5x^2 - 45y^2$
21. $9x^4 - 4$
22. $16x^4 - y^2$

Answers: 1. Even: d, e, g, j, p, r   Odd: a, b, f, h, i, l, o, q   Neither: c, k, m, n

2a. $-.864$ 2b. 1.416 3a. $y = -x - 3$ 3b. $y = -\frac{1}{3}x - \frac{1}{6}$

4a. $x = \dfrac{\ln 5}{2}$ 4b. $\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$ 4c. $x = e^4 - 3$ 5. $\cancel{3x^2 + 3xh + h^2}$ $\;2x + h + 2$

6. a. and b.
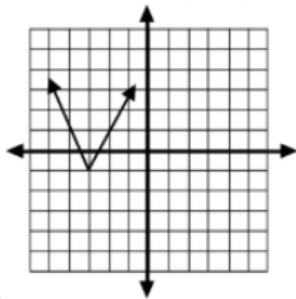
7a. $D:(-\infty, 3)\;\; R:(0,3]$

7b. $D:\mathbb{R}, x \neq 0\;\; R:[-.21723, 1)$

7c. $D:\mathbb{R}, x \neq 3\;\; R:\mathbb{R}, y \neq 7$

7d. $D:\mathbb{R}, x \neq 3, -5\;\; R:\mathbb{R}, y \neq 7/8$

7e. $D:(3,\infty)\;\; R:(-\infty,\infty)$

7f. Graph are the same except $g(x)$ does not have a hole at $(3,7)$ and $f(x)$ does.

8a. $2\ln x + \dfrac{1}{2}\ln y$ 8b. $\log_3(x+3) - 2\log_3 x$ 9a. Amp=4   9b. Period=4   9d. $y = -4x + 6$

10a. reflect $f(x)$ over x-axis   10b. reflect $f(x)$ over y-axis

11a. $\left(-3, \dfrac{5}{3}\right)$ 11b. $x = 3$ 11c. $y = 2$ 11d. $(2,0)$ 11e. $\left(0, \dfrac{4}{3}\right)$

12a. $V(x) = \dfrac{1}{4}(100x - 2x^3)$ 12b. $(0, 7.071)$ 12c. $(0, 68.041]$

13a. $x^2 + 5x + 25,\; x \neq 5$ 13b. $\dfrac{x^2 - 3x + 9}{x - 3},\; x \neq 3, -3$ 13c. $\dfrac{-1}{3(x+3)},\; x \neq 0, -3$ 13d. $2 + \sqrt{4 - x},\; x \neq 0$

14. $x = 10\sqrt{2} - 7$ 15. $V(h) = \dfrac{25\pi h^3}{1083}$ 16a. $18x^2 - 102x + 152$ 16b. 19 16c. $f^{-1}(x) = \dfrac{x+5}{3}$

17a. Runner B / the race was 13 miles for Runner A   17b. Runner A runs $\dfrac{24}{11}$ mph faster than Runner B

17c. 70.714 minutes   17d. 3 miles

18a. $\dfrac{\sqrt{3}}{2}$ 18b. $-\dfrac{\sqrt{2}}{2}$ 18c. $-\dfrac{\sqrt{3}}{3}$ 18d. -2 ~~19a. $\cos^2 x$ 19b. $2\sin x$~~

Answers for Q19 and Q20 are on the next page.

19. Factoring Trinomials

1. $(2x+1)(x-3)$
2. $(3x-2)(x+4)$
3. $(2y+1)(y+7)$
4. $(7a-4)(a-1)$
5. $(5n+2)(n+3)$
6. $(2y+3)(2y+1)$
7. $(3x+7)(x-1)$
8. $(2x+3)(x+5)$
9. $(3y-2)(3y+4)$
10. $(3x+4)(2x-5)$
11. $(2n-7)(n+2)$
12. prime
13. $(2x+5)(5x-6)$
14. $(3y+1)(4y+1)$
15. $(2n-1)(n+5)$
16. $(2x+3)(x+2)$
17. $(5a+3)(a-9)$
18. $(3x-8)(5x+4)$
19. $(2a-1)(4a-3)$
20. $(2y+5)(y-4)$

20. Factoring the difference of two squares

1. $(x+1)(x-1)$
2. $(x+3)(x-3)$
3. prime
4. $(x+5)(x-5)$
5. $(3y+4)(3y-4)$
6. $(2x+5)(2x-5)$
7. $(3x+1)(3x-1)$
8. $(a+x)(a-x)$
9. $(5+m)(5-m)$
10. $(x+4y)(x-4y)$
11. $(5m+n)(5m-n)$
12. $(4+x)(4-x)$
13. $(6m+11)(6m-11)$
14. $2(x+2)(x-2)$
15. prime
16. $(2a+9b)(2a-9b)$
17. $3(2x+5)(2x-5)$
18. $b(a+b)(a-b)$
19. $-2(7+x)(7-x)$ or $2(x+7)(x-7)$
20. $5(x+3y)(x-3y)$
21. $(3x^2+2)(3x^2-2)$
22. $(4x^2+y)(4x^2-y)$